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Recent Advances in Loop Quantum Cosmology

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Outline

- Purpose of Loop Quantum Gravity
- Introduction to Loop Quantum Cosmology
- Alternative Dynamics of LQC
- Effective Scenarios of LQC
- Other Advances in LQC

1. Purpose of Loop Quantum Gravity

- Motivations of Quantum Gravity

  From the beginning of last century to now, two fundamental theories of physics, QM and GR, have destroyed the coherent pictures of the physical world.

  - *Classical Gravity - Quantum Matter Inconsistency*

$$R_{\alpha\beta}[g] - \frac{1}{2}R[g]g_{\alpha\beta} = \kappa T_{\alpha\beta}[g]. \tag{1}$$

  - *Singularity in General Relativity*
  - *Infinity in Quantum Field Theory*
  - *The interpretation of black hole thermodynamics*

$$S_{BH} = Ar_{BH}/4\hbar. \tag{2}$$

- The Basic Ideas of LQG

  ⋆ Combine the basic principles of GR and QM.

  ⋆ The choice of the algebra of field functions to be quantized:

  Not the positive and negative components of the field modes as in conventional QFT; but the holonomies of the gravitational connection and the electric flux.

  ⋆ The physical motivation of using holonomies:

  Background independence.

  The relevant variables do not refer to what happens at a point, but rather refer to the relation between different points connected by a line,

$$A(c) = \mathcal{P} \exp\big(-\int_0^1 [A_a^i \dot{c}^a \tau_i]\, dt\big). \tag{3}$$

- Classical Connection Dynamics of GR

  – GR can be cast into a connection dynamical formalism on a spatial 3-manifold Σ [Ashtekar 1986, Barbero 1995], where the configuration is a $su(2)$ connection A_a^i and conjugate momentum is a densitized triad E_i^a.

  – The Hamiltonian density $\mathcal{H}_{tot}$ is a linear combination of first-class constraints.

- The Kinematical Framework of LQG
  - The kinematical Hilbert space: $\mathcal{H}_{kin} = L^2(\overline{\mathcal{A}},\ d\mu^0)$ is well defined on the space $\overline{\mathcal{A}}$ of generalized connections.
  - Uniqueness Theorem [LOST, 2005].
  - Geometric operators with discrete spectrum:

    Area operator [Rovelli and Smolin, 1995; Ashtekar and Lewandowski,1997]; Volume operator [Ashtekar and Lewandowski, 1995, 1997; Rovelli and Smolin, 1995]; Length operator [Thiemann 1998; YM, Soo, Yang, 2010]; $\hat{Q}$ operator [YM and Ling, 2000].
- The Quantum Dynamical Issues
  - Both Gaussian constraint and spatial diffeomorphism constraint are successfully implemented at quantum level [ALMMT, 1995].
  - Hamiltonian constraint operators can be well defined in $\mathcal{H}_{kin}$ or $\mathcal{H}^G$ [Thiemann 1997].

    Master constraint operators can be well defined in $\mathcal{H}_{Diff}$ [Thiemann, 2003, 2005; Dittrich and Thiemann, 2004; Han and YM, 2005].

2. Introduction to Loop Quantum Cosmology

- The idea that one should view holonomies rather than connections as basic variables for the quantization of gravity is successfully carried on in the symmetry-reduced models, known as Loop Quantum Cosmology.
- One freezes all but a finite number of degrees of freedom by imposing symmetries.

  The simplified framework provides a simple arena to test ideas and constructions.
- Symmetries: homogeneity and (or) isotropy.
- Example: Spatially flat FRW universe
  - Spatial 3-manifold: $\mathbb{R}^3$
  - Isometry: Euclidean group

- The Kinematical Setting of LQC
  - One has to introduce an elementary cell $\mathcal{V}$ and restricts all integrations to this cell.
  - Fix a fiducial flat metric ${}^o q_{ab}$ and denote by V_o the volume of $\mathcal{V}$ in this geometry.

    The gravitational phase space variables —the connections and the density weighted triads — can be expressed as

    $A_a^i = c\, V_o^{-(1/3)}\ {}^o\omega_a^i$ and $E_i^a = p\, V_o^{-(2/3)} \sqrt{{}^o q}\ {}^o e_i^a$,

    where $({}^o\omega_a^i, {}^o e_i^a)$ are a set of orthonormal co-triads and triads compatible with ${}^o q_{ab}$ and adapted to $\mathcal{V}$.
  - p is related to the scale factor a via $|p| = V_o^{2/3} a^2$.
  - The fundamental Poisson bracket is given by: $\{c,\, p\} = \kappa\gamma/3$,

    where $\kappa = 8\pi G$.
  - The gravitational part of the Hamiltonian constraint reads

    $C_{\rm grav} = -6c^2\sqrt{|p|}/\gamma^2$.

- To pass to the quantum theory, one constructs a kinematical Hilbert space $\mathcal{H}_{\rm kin}^{\rm grav} = L^2(\mathbb{R}_{\rm Bohr}, {\rm d}\mu_{\rm Bohr})$, where $\mathbb{R}_{\rm Bohr}$ is the Bohr compactification of the real line and ${\rm d}\mu_{\rm Bohr}$ is the Haar measure on it.
- There exists no operator corresponding to c, while holonomy operators are well defined.
- The Improved Scheme:
  - It is convenient to introduce new conjugate variables by a canonical transformation:

$$b := \frac{\sqrt{\Delta}}{2}\frac{c}{\sqrt{|p|}}, \qquad \nu := \frac{4}{3\sqrt{\Delta}} sgn(p)|p|^{\frac{3}{2}},$$

    where Δ $(\sim\ 4\sqrt{3}\pi\gamma\, l_p^2)$ is the smallest non-zero eigenvalue of area operator in full LQG.
  - In the kinematical Hilbert space $\mathcal{H}_{\rm kin}^{\rm grav}$, eigenstates of $\hat{\nu}$, which are labeled by real numbers v, constitute an orthonormal basis as: $\langle v_1|v_2\rangle = \delta_{v_1,v_2}$.
  - The fundamental operators act on $|v\rangle$ as: $\hat{\nu}\,|v\rangle = (8\pi\gamma l_p^2/3)v|v\rangle$ and $\widehat{e^{ib}}\,|v\rangle = |v+1\rangle$.

3. Alternative Dynamics for LQC

- APS Dynamics
  - The gravitational part of the APS Hamiltonian operator was given in the v representation by [Ashtekar, Pawlowski, Singh, 2006]:

$$\hat{C}_{\text{grav}}\,|v\rangle = f_+(v)|v+4\rangle + f_o(v)|v\rangle + f_-(v)|v-4\rangle. \tag{4}$$

  - To identify a dynamical matter field as an internal clock, one takes a massless scalar field ϕ with Hamiltonian $C_\phi = |p|^{-\frac{3}{2}} p_\phi^2/2$, where p_ϕ denotes the momentum of ϕ.

- Alternative Dynamics
  - LQC Gravitational Hamiltonian operator with Lorentz and Euclidean terms [Yang, Ding, YM, 2009]:

$$\begin{aligned}\hat{H}^F_{\text{grav}}|v\rangle = \;& F'_+(v)|v+8\rangle + f'_+(v)|v+4\rangle + \left(F'_o(v) + f'_o(v)\right)|v\rangle \\ & + f'_-(v)|v-4\rangle + F'_-(v)|v-8\rangle.\end{aligned} \tag{5}$$

  - The new proposed Hamiltonian constraint operator $\hat{H}^F_{\text{grav}}$ contains more terms with step of different size comparing to the original APS Hamiltonian operator.

4. Effective Scenarios of LQC

- Effective Hamiltonian and Friedmann Equation
  - We can further obtain an effective Hamiltonian of $\hat{H}_F = \hat{H}^F_{\rm grav} + \hat{H}_\phi$ with the relevant quantum corrections of order $\epsilon^2, 1/v^2\epsilon^2, \hbar^2/\sigma^2 p_\phi^2$ as

$$\begin{aligned}\mathcal{H}^F_{\rm eff} &= -\tfrac{3^2\sqrt{6}}{2^3}\tfrac{\hbar^{1/2}}{\gamma^{3/2}\kappa^{1/2}} L\,|v|\big(\sin^2(2b)\left(1-(1+\gamma^2)\sin^2(2b)\right)+2\epsilon^2\big)\\ &\quad + \left(\tfrac{\kappa\gamma\hbar}{6}\right)^{3/2}\tfrac{|v|}{L}\rho\left(1+\tfrac{1}{2|v|^2\epsilon^2}+\tfrac{\hbar^2}{2\sigma^2p_\phi^2}\right), \qquad (6)\end{aligned}$$

where $\rho = \frac{1}{2}\left(\frac{6}{\kappa\gamma\hbar}\right)^3\left(\frac{L}{|v|}\right)^2 p_\phi^2$ is the density of the matter field.

  - The modified Friedmann equation can then be derived as:

$$H_F^2 = \left(\frac{\dot{v}}{3v}\right)^2 = \frac{\kappa}{3}\frac{\rho_c}{4(1+\gamma^2)^2}\left(1-\sqrt{1-\chi_F}\right)\left(1+2\gamma^2+\sqrt{1-\chi_F}\right)(1-\chi_F)\,, \qquad (7)$$

where

$$\chi_F = 4(1+\gamma^2)(\frac{\rho}{\rho_c}\left(1+\frac{1}{2|v|^2\epsilon^2}+\frac{\hbar^2}{2\sigma^2p_\phi^2}\right)-2\epsilon^2). \qquad (8)$$

- Effective Scenarios
  - In the leading order effective description, when energy density of the scalar field reaches to the critical energy density $\rho_c^F = \rho_c/4(1+\gamma^2)$, the universe bounces from the contracting branch to the expanding branch.
  - Moreover, the minus sign in front of the ϵ^2 term in the expression (8) of χ_F may lead to a qualitatively different scenario from the leading order effective theory.
  - The concrete form of quantum fluctuations or the Gaussian spread ϵ plays a key role here.
  - A simple setting could be $\epsilon = \lambda(r)v^{-r(\phi)}$, where $0 \leq r(\phi) \leq 1$ and the parameter $\lambda(r)$ has to be suitably chosen for different value of r.
  - For $r = 0$, besides the quantum bounce when the matter density ρ increases to the Planck scale, the universe would also undergo a recollapse when ρ decreases to $\rho_{\text{coll}}^F \approx 2\epsilon^2\rho_c$.

    It is easy to see from Eqs.(8) and (7) that an expanding universe would undergo the recollapse and become cyclic provided $0 \leq r < 1$ asymptotically.

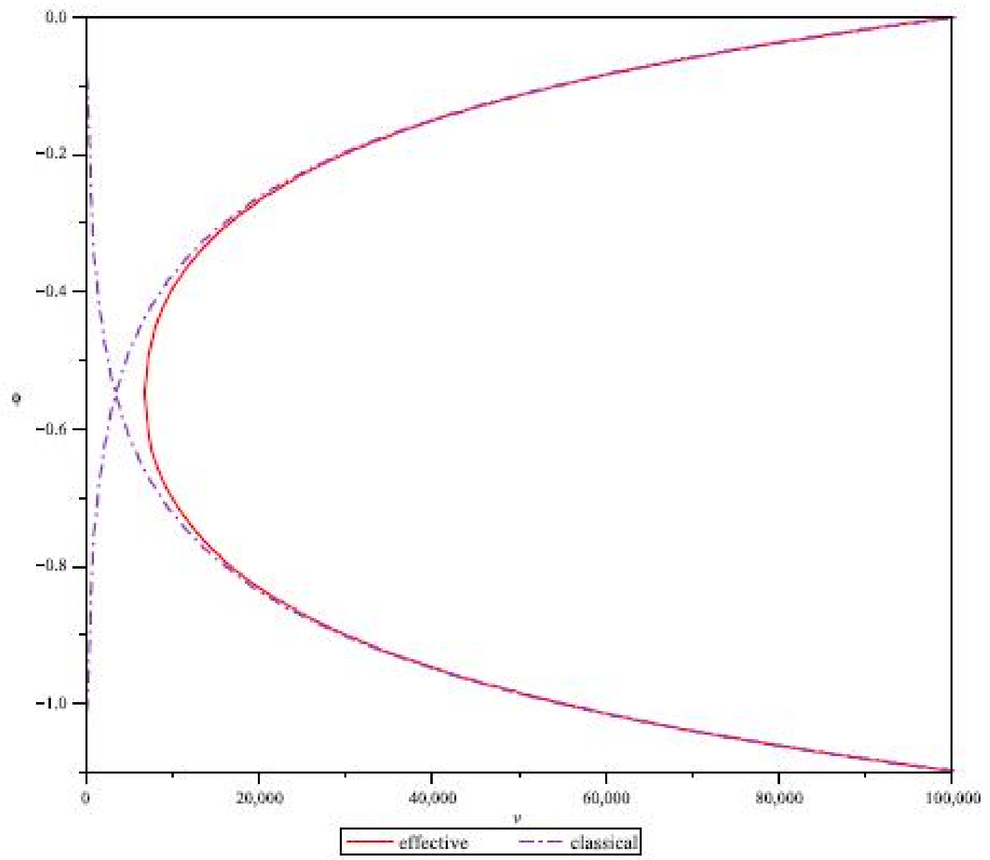

FIG. 1: The effective dynamics represented by the observable $v|_\phi$ are compared to classical trajectories. In this simulation, the parameters were: $G = \hbar = 1$, $p_\phi = 10\,000$, $\epsilon = 0.001$, $\sigma = 0.01$ with initial data $v_o = 100\,000$.

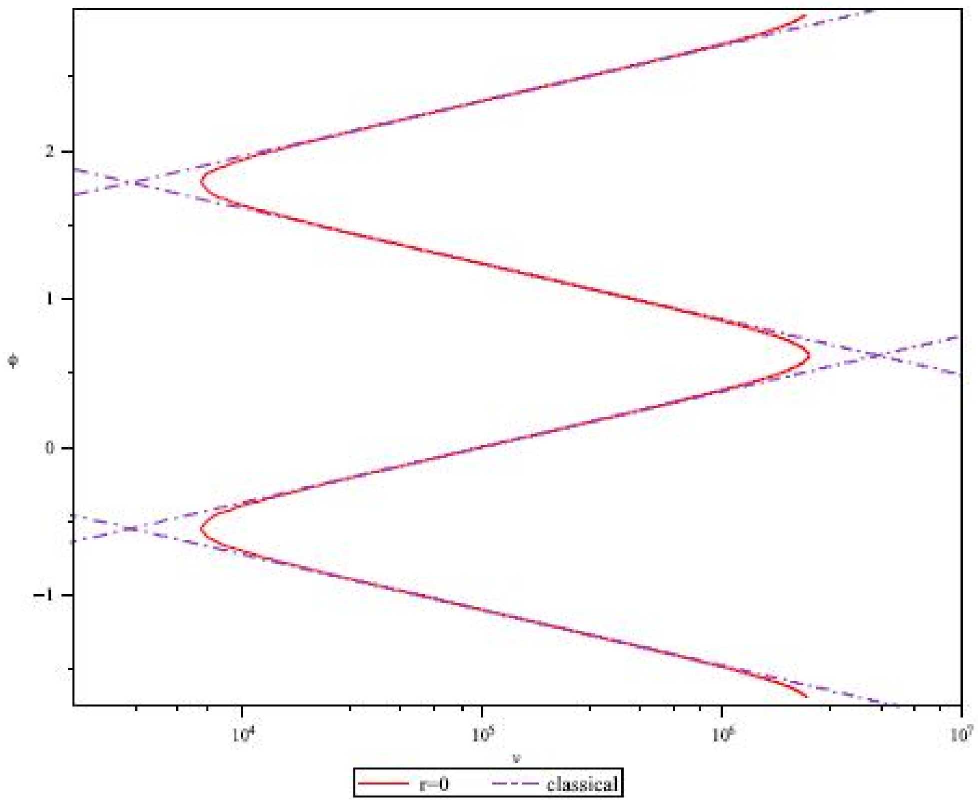

FIG. 2: The cyclic model is compared with expanding and contracting classical trajectories. In this simulation, the parameters were: $G = \hbar = 1$, $p_\phi = 10\,000$, $\epsilon = 0.001$, $\sigma = 0.01$ with initial data $v_o = 100\,000$.

5. Other Advances in LQC

- Inflation in LQC
  - The loop quantum cosmological modification of Friedmann equation implies a phase of super-inflation immediately after the bounce [Bojowald 2002].
  - For a wide class of potentials, the super-inflation funnels the phase space trajectories to initial conditions which virtually guarantee a slow roll inflation with 60 or more e-foldings [Ashtekar, Sloan, 2009, 2011].
- Path-integral formulation of LQC
  - Concrete evidence in support of the general paradigm underlying spin foam models is provided by LQC [Ashtekar, Campiglia, Henderson, 2009, 2010].
  - The vertex expansion of LQC path-integral is a convergent series [Henderson, Rovelli, Vidotto, Wilson-Ewing, 2010]
  - The first-order effective Hamiltonians of LQC can be derived from the path-integral formalism [Ashtekar, Campiglia, Henderson, 2010; Huang, Ma, Qin, 2011]
- Phenomenological issues

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Thank you!