Review of Dark Matter Models

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I. Introduction

Based on numerous cosmological and astrophysical studies, we believe that our Universe is flat and the total energy density is critical

$$\rho_{\rm tot} \simeq \rho_c \equiv \frac{3H_0^2}{8\pi G_{\rm N}} \; , \; \Omega_{\rm tot} \equiv \frac{\rho_{\rm tot}}{\rho_c} \simeq 100\% \; .$$

$$\Omega_\Lambda \simeq 72\% \; , \; \Omega_{\rm dm} \simeq 23\% \; , \; \Omega_{\rm b} \simeq 4.6\% \; ,$$
$$\Omega_\gamma \simeq 0.005\% \; , \; 0.1\% \leq \Omega_\nu \leq 1.5\% \; .$$

Conditions for Dark Matters:

- $SU(3)_C \times U(1)_{\rm EM}$ singlets.
- Stable due to additional symmetry, or lifetime is larger than the age of the Universe today t_0 about 13.7 Gyr

$$\tau_{\rm dm} > t_0 \simeq 4.3 \times 10^{17}\ {\rm s}\ .$$

- Energy Density $\Omega_{\rm dm} \simeq 0.11$.

The SM does not have such kind of particles.

Stability for the SM Particles:

- Photon: massless $U(1)_{EM}$ gauge boson.
- Electron: lightest particles charged under $U(1)_{EM}$.
- Lightest Neutrino: lightest fermion or Lorentz invariance.
- Proton: accidental baryon symmetry, and dimension-5/6 in GUTs.

$$\mathcal{L} = \frac{g_{23}^2 \epsilon^{ijk}}{2M_{32}^2} \left[((\bar{d}_k^c \cos\theta_c + \bar{s}_k^c \sin\theta_c)\gamma^\mu P_L u_j) \times (u_i \gamma_\mu P_L e_L) + h.c. \right] .$$

The DM stability.

Energy Density:

- Thermal: WIMP
- Non-thermal: cosmic string, freeze-in mechanism, etc.
- Multiple Component Dark Matter
- Superstring Cosmology: Dilution

Philosophy for Dark Matter Models:

- The simplest Model.
- The SM has some problems, and the dark matter is a particle in the extension of the SM that can solve these problem.
- Experiment inspired models.

The Simplest Model

A real Klein–Gordon field S with a Z_2 parity.

$$\mathcal{L}_S = \frac{1}{2}\partial_\mu S \partial^\mu S - \frac{1}{2} m_S^2 S^2 - \frac{k}{2}|H|^2 S^2 - \frac{h}{4!} S^4 \, .$$

To be consistent with the triviality and stability bounds, we find $m_S \simeq 5.5$ GeV–1.8 TeV.

Major Problems in the SM

- Fine-Tuning Problems

- Aesthetic Problems

Fine-Tuning Problems:

- Cosmological constant problem

$$\Lambda_{\rm CC} \sim 10^{-122} M_{\rm Pl}^4 \ .$$

- Gauge hierarchy problem

$$M_{\rm EW} \sim 10^{-16} M_{\rm Pl} \ .$$

- Strong CP probelm

$$\theta < 10^{-9} \ .$$

- The SM fermion masses and mixings

$$m_{\rm electron} \sim 10^{-5} m_{\rm top} \ .$$

Aesthetic Problems:

- Interaction unification
- Fermion unification
- Gauge coupling unification
- Charge quantization

The first two prolems can be solved when we embed the SM into the Grand Unified Theories (GUTs) and string models.

II. Natural DM Models

Axion

$$\mathcal{L} \;=\; \theta \frac{g_s^2}{32\pi^2} G^a_{\mu\nu} \widetilde{G}^{a\mu\nu} \; .$$

This term violates the discrete symmetries P, T and CP for any value of $\theta \neq n\pi$.

Strong CP Problem

- $\overline{\theta} = \theta + \theta_{\rm q}$ parameter is a dimensionless coupling constant which is infinitely renormalized by radiative corrections.

- No theoretical reason for $\overline{\theta}$ as small as 10^{-9} required by the experimental bound on the EDM of the neutron.

$$d_{\rm N} \sim \frac{e\overline{\theta}}{m_{\rm N}(m_u^{-1} + m_d^{-1} + m_s^{-1})} \sim 3 \times 10^{-16}\overline{\theta}\ {\rm e-cm} \quad \sim 6 \times 10^{-25}\ {\rm e-cm}\ .$$

Peccei–Quinn Mechanism

- $\overline{\theta} = \theta + \theta_{\rm q} + a/f_a$

$$V_{\rm Instanton} \simeq \Lambda^4_{QCD} \left(1 - \cos \overline{\theta}\right) \, .$$

- $10^{10}\ {\rm GeV} \ < \ f_a \ < \ 10^{12}\ {\rm GeV}$
- Axion can be a cold dark matter candidate.
- The axion solution can be stabilized by the gauged discrete PQ symmetry [a].

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[a] Babu, Gogoladze, Wang; Barger, Chiang, Jiang, TL

Axion Dark Matter

Axion lifetime

$$\mathcal{L}_{a\gamma\gamma} \;=\; \frac{g_{a\gamma\gamma}}{4} a F^{a}_{\mu\nu} \widetilde{F}^{a\mu\nu} \; .$$

$$\tau_a = \Gamma^{-1}_{a\to\gamma\gamma} = \frac{64\pi}{g^2_{a\gamma\gamma} m_a^3} \; .$$

For $\tau_a > t_0$, we obtain $f_a > 3 \times 10^5$ GeV.

Coincidence: axion relic density

$$\Omega_a h^2 \sim \left(\frac{f_a}{10^{12}\mathrm{GeV}}\right)^{\frac{7}{6}} \left(\frac{200\mathrm{MeV}}{\Lambda_{\mathrm{QCD}}}\right)^{\frac{3}{4}} \ .$$

$$\Omega_D h^2 \sim 0.11 \ .$$

$$f_a \sim 10^{11}\mathrm{GeV} \ , \quad m_a \sim 10^{-5}\mathrm{eV} \ .$$

Supersymmetric Standard Models

- Solving the gauge hierarchy problem
- Gauge coupling unification
- Radiatively electroweak symmetry breaking

  Large top quark mass
- Natural dark matter candidates

  Neutralino, sneutrino, gravitino, axino, ...
- Electroweak baryogenesis
- Electroweak precision: R parity

R-Parity:

$$R = (-1)^{3(B-L)+2S} \ .$$

The SM particles are even while their superpartners are odd under R parity, thus, the lighest supersymmetric particles (LSP) are can not decay.

Dark Matter: Neutralino

In the baisis $(\widetilde{B}^0, \widetilde{W}^0, \widetilde{H}_1^0, \widetilde{H}_2^0)$, the neutralino mass matrix is:

$$Q_1 = \begin{pmatrix} M_1 & 0 & -M_z \sin\theta_W \cos\beta & M_z \sin\theta_W \sin\beta \\ 0 & M_2 & M_z \cos\theta_W \cos\beta & -M_z \cos\theta_W \sin\beta \\ -M_z \sin\theta_W \cos\beta & M_z \cos\theta_W \cos\beta & 0 & -\mu \\ M_z \sin\theta_W \sin\beta & -M_z \cos\theta_W \sin\beta & -\mu & 0 \end{pmatrix} .$$

The lightest neutralino is the dark matter condidate.

Viable Parameter Spaces: $\Omega_{\chi_1^0}$ exceeds $\Omega_{\rm dm}$ in most of parameter spaces, so, the parameter spaces for $\Omega_{\chi_1^0} \simeq \Omega_{\rm dm}$ are special

- Light slepton region: $\chi_1^0\chi_1^0 \to l^+l^-$ via slepton exchange. Problem, tension with the LEP Higgs mass bound.

- Light Focus point region/hyperbolic branch: χ_1^0 has a significant Higgsino admixture so that $\chi_1^0\chi_1^0 \to W^+W^-$ and Z^0Z^0 become efficient.

- Coannihilation region: the NLSP has a mass very close to χ_1^0, and its number density during the freeze out of χ_1^0 is still sizable. $\chi_1^0 - \tilde{\tau}_1$ coannihilation process.

- Higgs funnel: $2m_{\chi_1^0}$ is close to the CP odd Higgs A^0 mass. $\chi_1^0\chi_1^0 \rightarrow A^0 \rightarrow b\bar{b}$

III. Typical Models

KK dark matter:

- Universal extra dimensions
- Symmetry: KK parity conservation, even or odd KK level
- Dark matter candidate: B^1

$SO(10)$ **Model or** $U(1)_{B-L}$ **Model:**

- $U(1)_{B-L}$ is broken down to a Z_2 symmetry
- Z_2 odd scalars: 16, 144, 560
- Z_2 even fermion: 10, 45, 120, 126, 210

Minimal DM:

- $SU(2)_L$ high representation: neutral component without tree-level interactions to Z.
- Fermions: quintuplet, septuplet, ...
- Scalars: Septuplet, nonuplet, ...

Mirror Dark matter:

- Mirror sector has the same gauge interactions and particle content as the observable sector. Only gravity interaction between them.
- For the BBN, the temperature in the Mirror sector sector is low than the observable sector.
- The two sectors have different initial conditions, do not come into thermal equilibrium at the later epoch, evolve independently, conserve their own entropies, and maintain approximately constant ration among their temperatures.

Degenerated dark matter candidates:

- The INTEGRAL experiment detects a 511 KeV emission line from the galactic center, consistent with injection of a few MeV positrons. This can be explained from the dark matter annihilation if the mass splittings are a few MeVs.
- The DAMA signal, which is still compatible with the null results of the other DM experiments within the framework of inelastic DM with splitting about 100 KeV.

Models:

$$
\begin{aligned}
\mathcal{L} &= \frac{1}{2}\partial_\mu\phi\partial^\mu\phi + \chi_i^\dagger\sigma_\mu\partial^\mu\chi_i - m_D\chi_1\chi_2 \\
&\quad -\lambda_1\phi\chi_1\chi_1 - \lambda_2\phi\chi_2\chi_2 - V(\phi)\ .
\end{aligned}
$$

Z_4 symmetry:

$$
\chi_{1,2} \longrightarrow e^{\pm i\pi/2}\chi_{1,2}\ ,\ \phi \longrightarrow -\phi\ .
$$

In the basis $\chi, \chi_* = 1/\sqrt{2}(\chi_1 \mp \chi_2)$, the mass matrix for χ's is

$$M = \begin{pmatrix} \lambda_+\phi - m_D & \lambda_-\phi \\ \lambda_-\phi & \lambda_+\phi + m_D \end{pmatrix} ,$$

where $\lambda_\pm = \frac{1}{2}(\lambda_1 \pm \lambda_2)$.

There are simple conclusions to draw from this expression. At leading order, the Dirac fermion is understood as two degenerate Majorana fermions. If the $\mathbf{Z}_4$ symmetry is broken weakly to $\mathbf{Z}_2$, for instance by a ϕ expectation value, we expect these states to be split by a small amount $\delta = 2\lambda_+\langle\phi\rangle$.

Higgs mechanism for splitting

$$V(\phi) = -\frac{m_\phi^2}{2}\phi^2 + \frac{\kappa}{4}\phi^4 \ ,$$

from which we yield a vacuum expectation value (vev) $\langle\phi^2\rangle = m_\phi^2/\kappa$. The mass splitting is then just $\delta = \lambda_+ m_\phi/\sqrt{\kappa}$.

Hidden vector dark matter:

- Hidden sector has a $SU(2)_{HS}$ gauge symmetry, and a doublet ϕ

$$\begin{aligned}\mathcal{L} &= \mathcal{L}^{SM} - \frac{1}{4}F'^{\mu\nu} \cdot F'_{\mu\nu} + (D_\mu\phi)^\dagger(D^\mu\phi) - \lambda_m \phi^\dagger\phi H^\dagger H \\ &\quad -\mu_\phi^2\phi^\dagger\phi - \lambda_\phi(\phi^\dagger\phi)^2 \ .\end{aligned}$$

- If $SU(2)_{HS}$ is spontaneously broken (i.e. $\mu_\phi^2 < 0$), defining $\phi \equiv exp(i\tau \cdot \xi/v_\phi) \cdot (0, \frac{1}{\sqrt{2}}[v_\phi + \eta'])^T$, and gauge rotating away the ξ part to absorb it in $A_\mu = UA'_\mu U^{-1} - \frac{i}{g}[\partial_\mu U]U^{-1}$ with $U = exp(-i\tau \cdot \xi/v_\phi)$

$$\begin{aligned}\mathcal{L} &= \mathcal{L}_{SM} - \frac{1}{4}F_{\mu\nu}\cdot F^{\mu\nu} + \frac{1}{8}(g_\phi v_\phi)^2 A_\mu \cdot A^\mu + \frac{1}{8}g_\phi^2 A_\mu \cdot A^\mu \eta'^2 \\ &+\frac{1}{4}g_\phi^2 v_\phi A_\mu \cdot A^\mu \eta' + \frac{1}{2}(\partial_\mu \eta')^2 - \frac{\lambda_m}{2}(\eta' + v_\phi)^2 H^\dagger H \\ &-\frac{\mu_\phi^2}{2}(\eta' + v_\phi)^2 - \frac{\lambda_\phi}{4}(\eta' + v_\phi)^4 ,\end{aligned}$$

which gives $m_A = g_\phi v_\phi/2$ and $m_{\eta'}^2 = -2\mu_\phi^2$.

The Lagrangian above has an important property: it displays a $SO(3)$ custodial symmetry in the A_μ^i component space. As a result the 3 A_i^μ components are degenerate in mass and are stable. Any decay to $SO(3)$ singlets (as η') is forbidden by the custodial symmetry.

V. Experimental Inspired Models

PAMELA, ATIC, or Fermi-LAT

- Boost Factor: 1000
- Mass: 1000 GeV
- Annilation or decay dominant into leptons.

Solutions:

- Non-thermal DM production [a].
- Sommerfeld enhancement [b].
- Breit-Wigner enhancement: the resonent mass is just below the twice dark matter mass [c].
- Decay DM: similar to proton decay or couplings to the neutrino mass terms $SSLLH_uH_u$.

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[a] R. Jeannerot, X. Zhang and R. Branderberger.

[b] N. Arkani-Hamed, D. P. Finkbeiner, T. Slatyer and N. Weiner.

[c] M. Ibe, H. Murayama and T. T. Yanagida; W. L. Guo and Y. L. Wu.

CoGENT, DAMA, CRESST experiments:

- Light DM: 10 GeV
- DAMA: $\sigma_N \sim 2 \times 10^{-4}$ pb
- CoGENT: $\sigma_N \sim 5 \times 10^{-5}$ pb
- XENON and CDMS: Null results

DM mass:

- Asymmetric dark matter: $M_{DM} \sim m_p \times \Omega_{\rm dm}/\Omega_{\rm b} = 5$ GeV
- Supersymmetric SM with a Singlet S

Cross Section: $-0.74 \leq f_n/f_p \leq -0.63$

$$\sigma_A \; = \; \frac{\mu_A^2}{M_*^4} \left[f_p Z + f_n (A - Z) \right]^2 \; .$$

- Isospin Violating DM [a]
- $U(1)'$ Model [b]
- Vector-Like Particle Model [c]
- CRESST Experiment: $f_n/f_p \sim -0.7$, and XENON
- Problem: CDMS II.

[a] Chang, Liu, Pierce, Weiner, Yavin.
[b] Kang, TL, Liu, Tong, Yang.
[c] Feng, Kumar, Marfatia, Sanford.

Thank you very much!